



Supporting Information “SI” for
A mass-balance approach for predicting lake phosphorus
concentrations as a function of external phosphorus
loading: Application to the Lake St. Clair – Lake Erie
System (Canada – USA).

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Table S-1. Lake Erie total phosphorus (TP) targets and observed/target/expected spring offshore TP concentrations.

Segment/Basin	1978 GLWQA		2016 GLWQA		2003-2016 conditions		New conditions	
	Target TP load (MTA)	Target TP concentration ($\mu\text{g L}^{-1}$)	New target TP load (MTA)	Expected TP concentration ($\mu\text{g L}^{-1}$)	Mean TP load (MTA)	Observed TP concentration ($\mu\text{g L}^{-1}$)	Target TP load (MTA)	Predicted TP concentration ($\mu\text{g L}^{-1}$)
Lake Erie, including:	11,000		~6600		11,700		~6600	
Western Basin		15.0		12.0		23.1		13.1
Central Basin		10.0		6.0		13.0		7.8
Eastern Basin		10.0		6.0		9.6		5.9

Table S-2. Mean values (Mean), standard deviation (SD), maximum (Max) and minimum (Min) values of the water and total phosphorus (TP) balance components used in this study. The original data are presented in Bocaniov et al. (2023). “Total In” stands for a total load.

System Component	Flux (ID#)*	Annual Water Load (km ³)				Annual Total Phosphorus (TP) Load (MTA)			
		Mean	SD	Max	Min	Mean	SD	Max	Min
1. SCR	1	159.1	12.1	190.9	147.3	1862.3	327.3	2633.0	1533.8
	2	0.8	0.3	1.5	0.3	207.9	106.9	471.9	71.2
	Total In	160.0	12.0	191.7	148.2	2070.2	298.3	2796.2	1739.9
2. LSC	1	0.2	0.0	0.2	0.2	-	-	-	-
	2	4.5	1.2	7.0	2.3	911.3	252.3	1434.7	514.5
	3	0.8	0.1	1.0	0.6	52.1	37.1	153.5	16.6
	4	0.02	0.0	0.02	0.02	0.3	0.0	0.3	0.3
	5	5.6	1.2	8.1	3.2	963.8	257.6	1494.4	547.0
	6	159.9	12.0	191.7	148.2	2070.2	298.3	2796.2	1739.9
Total In	165.5	12.0	196.6	153.9	3033.5	358.1	3670.1	2371.2	
Evaporation	1.0	0.04	1.0	0.9	-	-	-	-	
3. DR	2	0.7	0.1	0.9	0.4	665.5	152.9	850.6	457.7
	6	164.5	12.0	196.2	151.6	2446.9	274.0	2943.7	1983.5
	Total In	165.2	12.0	196.9	152.3	3112.4	407.5	3726.3	2441.2
4. WB	2	9.1	1.68	12.7	6.6	3019.7	836.7	4645.8	1709.8
	3	2.6	0.4	3.3	1.8	127.2	47.3	235.2	66.0
	4	0.05	0.0	0.05	0.05	0.9	0.0	0.9	0.9
	5	11.7	1.8	15.3	9.4	3147.8	840.0	4750.8	1888.1
	6	165.2	12.0	196.9	152.3	3112.4	407.5	3726.3	2441.2
	Total In	176.9	11.6	206.3	162.4	6260.3	990.7	8093.8	4721.7
Evaporation	2.9	0.3	3.4	2.5	-	-	-	-	

*Flux ID numbers are defined in Figure 2.

Table S-2. (Continued).

System Component	Flux (ID#)*	Water Load (km ³)				Total Phosphorus (TP) Load (MTA)			
		Mean	SD	Max	Min	Mean	SD	Max	Min
5. CB	2	8.5	1.8	11.3	5.3	2538.2	731.1	3881.5	1358.9
	3	12.9	2.0	16.4	8.9	624.0	231.1	1155.8	324.1
	4	0.2	0.0	0.2	0.2	4.5	0.0	4.5	4.5
	5	21.6	2.2	26.3	18.8	3166.8	763.3	4491.9	1882.5
	6	174.0	11.6	203.5	158.8	3363.8	984.7	6031.4	2044.7
	7	-	-	-	-	199.4	130.8	519.8	65.6
	Total In	195.6	11.3	223.6	179.5	6730.0	1355.4	9799.2	4484.1
Evaporati	14.2	1.3	16.7	12.5	-	-	-	-	
6. EB	2	7.6	1.5	10.5	5.5	1454.7	540.6	2661.1	830.7
	3	5.0	0.8	6.3	3.4	241.1	89.3	446.6	125.2
	4	0.1	0.0	0.1	0.1	1.8	0.0	1.8	1.8
	5	12.7	1.5	16.0	10.0	1697.5	570.0	3023.7	1121.9
	6	180.5	11.9	209.0	161.4	1872.2	330.3	2388.8	1304.2
	7	-	-	-	-	612.5	229.4	1875.5	1097.4
	Total In	193.1	11.8	220.3	173.8	4182.3	666.1	5660.1	3187.7
Evaporati	5.5	0.5	6.5	4.8	-	-	-	-	

*Flux ID numbers are defined in Figure 2.

Table S-3. Observed segment-specific total phosphorus (TP; $\mu\text{gP L}^{-1}$) concentrations* for Lake St. Clair and Lake Erie as reported in Bocaniov et al. (2023).

Water Year	Lake St. Clair	Lake Erie									
		Western Basin (WB)			Central Basin (CB)			Eastern Basin (EB)			
		Spring (April)	Summer (August)	Mean	Spring (April)	Summer (August)	Mean	Spring (April)	Summer (August)	Mean	Spring (April)
2003	17.3	36.5	10.7	23.6	18.4	6.6	12.5	8.7	5.2	6.9	
2004	16.3	16.6	15.8	16.2	9.8	5.3	7.6	11.1	4.3	7.7	
2005	16.2	17.5	15.3	16.4	15.1	5.9	10.5	8.2	3.7	6.0	
2006	16.4	37.6	11.9	24.8	11.5	6.9	9.2	10.3	4.6	7.5	
2007	16.2	22.7	12.6	17.7	18.4	7.6	13.0	14.1	4.7	9.4	
2008	16.0	14.9	9.3	12.1	14.6	7.9	11.3	9.1	5.2	7.2	
2009	17.4	55.8	12.1	33.9	11.5	9.4	10.4	8.5	4.6	6.6	
2010	15.4	13.1	16.2	14.6	12.9	10.0	11.4	10.0	4.1	7.0	
2011	14.6	27.1	22.0	24.5	12.2	6.0	9.1	6.1	4.9	5.5	
2012	14.6	7.3	33.5	20.4	8.4	11.2	9.8	10.0	4.3	7.1	
2013	13.1	24.5	16.6	20.6	18.1	7.8	13.0	12.0	6.7	9.4	
2014	12.1	9.7	19.5	14.6	7.9	6.9	7.4	9.4	5.4	7.4	
2015	11.6	20.9	10.8	15.9	9.0	9.6	9.3	6.4	5.9	6.1	
2016	12.1	19.2	13.7	16.4	14.6	7.6	11.1	10.8	5.7	8.3	
Aver.	14.95	23.12	15.72	19.42	13.03	7.76	10.39	9.62	4.95	7.29	

* For Lake St. Clair they were based on Lake St. Clair outflow concentration; and for Lake Erie they were calculated from the basin-specific offshore concentrations measured during spring (April) and summer (August) U.S. Environmental Protection Agency Great Lakes National Program Office (U.S. EPA GLNPO) water quality surveys. See Bocaniov et al. (2023) for more details.

Table S-4. Model equations for calculating the mean annual TP concentrations for six segments of Lake St. Clair – Lake Erie system. See Table S-5 for the definitions of the symbols.

I.D.#	Segment	Equation	Eq. #
1.1	St. Clair River (SCR)	$c^{SCR} = c_{in}^{SCR} \cdot \frac{R_p^{SCR(IN)}}{R_w^{SCR(IN)}}$	(9)
1.2		$c_{in}^{SCR} = \frac{W_t^{SCR}}{Q_t^{SCR}} = \frac{W_s^{LH} + W_w^{SCR}}{Q_{LH}^{LH} + Q_w^{SCR}}$	(10)
2.1	Lake St. Clair (LSC)	$c^{LSC} = c_{in}^{LSC} \cdot \frac{R_p^{LSC(IN)}}{R_w^{LSC(IN)}}$	(11)
2.2		$c_{in}^{LSC} = \frac{W_t^{LSC}}{Q_t^{LSC}} = \frac{W_s^{SCR} + W_w^{LSC} + W_{atm}^{LSC} + W_{gr}^{LSC}}{Q_s^{SCR} + Q_w^{SCR} + Q_{pr}^{SCR} + Q_{gr}^{SCR}}$	(12)
3.1	Detroit River (DR)	$c^{DR} = c_{in}^{DR} \cdot \frac{R_p^{DR(IN)}}{R_w^{DR(IN)}}$	(13)
3.2		$c_{in}^{DR} = \frac{W_t^{DR}}{Q_t^{DR}} = \frac{W_s^{LSC} + W_w^{LSC} + W_{atm}^{LSC} + W_{gr}^{LSC}}{Q_s^{LSC} + Q_w^{LSC} + Q_{pr}^{LSC} + Q_{gr}^{LSC}}$	(14)
4.1	Western Basin (WB)	$c^{WB} = c_{in}^{WB} \cdot \frac{R_p^{WB(IN)}}{R_w^{WB(IN)}}$	(15)
4.2		$c_{in}^{WB} = \frac{W_t^{WB}}{Q_t^{WB}} = \frac{W_s^{DR} + W_w^{WB} + W_{atm}^{WB} + W_{gr}^{WB}}{Q_s^{DR} + Q_w^{WB} + Q_{pr}^{WB} + Q_{gr}^{WB}}$	(16)
5.1	Central Basin (CB)	$c^{CB} = c_{in}^{CB} \cdot \frac{R_p^{CB(IN)}}{R_w^{CB(IN)}}$	(17)
5.2		$c_{in}^{CB} = \frac{W_t^{CB}}{Q_t^{CB}} = \frac{W_s^{WB/CB} + W_w^{CB} + W_{atm}^{CB} + W_{gr}^{CB}}{Q_s^{WB/CB} + Q_w^{CB} + Q_{pr}^{CB} + Q_{gr}^{CB}}$	(18)
6.1	Eastern Basin (EB)	$c^{EB} = c_{in}^{EB} \cdot \frac{R_p^{EB(IN)}}{R_w^{EB(IN)}}$	(19)
6.2		$c_{in}^{EB} = \frac{W_t^{EB}}{Q_t^{EB}} = \frac{W_s^{CB/EB} + W_w^{EB} + W_{atm}^{EB} + W_{gr}^{EB}}{Q_s^{CB/EB} + Q_w^{EB} + Q_{pr}^{EB} + Q_{gr}^{EB}}$	(20)

Table S-5. Definitions of notations, acronyms, and variables descriptions in the model equations shown in Tables S-4 and S-8, as well as in Figures S-1 and S-2.

#	Notations	Explanation (units)
1	TP	Total Phosphorus
2	MT	Metric Tonnes
3	MTA	Metric Tonnes per Annum
4	C^i	TP concentration in segment i (mg m^{-3} or $\mu\text{g L}^{-1}$)
5	C_{in}^i	Mean TP concentration of the inflow to segment i (mg m^{-3} or $\mu\text{g L}^{-1}$)
6	<i>LH, SCR, LSC</i>	Abbreviations for Lake Huron (LH), the St. Clair River (SCR), Lake St. Clair (LSC)
7	<i>DR, WB</i>	Abbreviations for the Detroit River (DR), Western Basin (WB)
8	<i>CB, EB</i>	Abbreviations for the Central Basin (CB) and the Eastern Basin (EB)
9	<i>WB/CB</i>	Abbreviation for the outflow/load from the western basin to the central basin
10	<i>CB/EB</i>	Abbreviation for the outflow/load from the central basin to the eastern basin
11	W_j^i	TP load from segment i with subscript j representing a sub-segment component (MTA)
12	Q_j^i	Water load to segment i with subscript j representing a sub-segment component j (km^3)
13	s	Segment
14	w	Direct watershed
15	atm	Over-lake atmospheric deposition
16	pr	Over-lake precipitation
17	gr	Direct groundwater discharge ($\text{km}^3 \text{ yr}^{-1}$)
18	t	Total water load or total TP load to the segment i
19	$R_p^{i(in)}$	TP residence time in segment i (days)
20	$R_w^{i(in)}$	Water residence time in segment i (days)
21	Chl-a	Chlorophyll-a ($\mu\text{g L}^{-1}$)
22	PP	Seasonal (May 1 – October 31) integrated primary production ($\text{g C m}^{-2} \text{ yr}^{-1}$)
23	SD	Secchi disk depth (m)
24	FP	Fish community production ($\text{kg ha}^{-1} \text{ yr}^{-1}$)
25	FB	Annual mean fish community standing biomass (kg ha^{-1})

Table S-6. Coefficients to derive spring and summer total phosphorus (TP) concentrations from the simulated mean (spring and summer) values for the three basins of Lake Erie:

Segment	Mean (April & August) TP concentration ($\mu\text{gP L}^{-1}$)	Spring (April) TP concentration ($\mu\text{gP L}^{-1}$)	Summer (August) TP concentration ($\mu\text{gP L}^{-1}$)
Western Basin	A	1.191 x A	0.809 x A
Central Basin	B	1.254 x B	0.746 x B
Eastern Basin	C	1.320 x C	0.680 x C

Note: A, B, and C are arbitrary values for illustration purposes only.

Table S-7. Summary of the responses of water quality indicators (Chlorophyll-a: Chl-a; Secchi disk Depth: SD; seasonal² Primary Production: PP; annual Fish community Production: FP; and, annual mean Fish community standing Biomass: FB) to total Lake Erie total phosphorus (TP) load reductions from the previous target load of 11,000 MTA (IJC, 1978) to a new target load of 6,600 MTA (GLWQA, 2016) derived from the load-response curves presented in Figures S-1 and S-2.

Water Quality Indicator	Units	Basin/Lake	Total Lake Erie Load (MTA)		Change	
			11,000	6,600	Absolute	Relative (as % change)
Chl-a ¹	$\mu\text{g L}^{-1}$	Western	3.5	2.2	-1.4	-38
		Central	2.2	1.4	-0.8	-35
		Eastern	1.7	1.1	-0.6	-35
SD ¹	m	Western	3.6	4.8	1.2	32
		Central	4.8	6.0	1.3	27
		Eastern	5.5	6.9	1.4	26
PP	$\text{gC m}^{-2} \text{yr}^{-1}$	Western	203	143	-61	-30
		Central	139	96	-43	-31
		Eastern	109	73	-35	-32
FP	$\text{kg ha}^{-1} \text{yr}^{-1}$	Western	11.3	8.3	-3	-27
		Central	8.3	6.4	-1.9	-23
		Eastern	7.1	5.5	-1.6	-23
	10^3 MTA yr^{-1}	Western	3.7	2.7	-1.0	-26.6
		Central	13.4	10.3	-3.1	-23.3
		Eastern	4.4	3.4	-1.0	-22.9
FB	kg ha^{-1}	Lake Erie	21.6	16.5	-5.1	-23.8
		Western	23.9	17.2	-5.7	-25
		Central	17.2	13.5	-3.7	-22
	10^3 MTA	Eastern	14.9	11.8	-3.2	-21
		Western	7.5	5.6	-1.9	-24.8
		Central	27.8	21.8	-6.0	-21.7
	Lake Erie	Eastern	9.6	7.5	-2.1	-22.7
		Lake Erie	44.6	34.7	-9.9	-22.1

¹ Summer values; ² May 1 to October 31.

Table S-8. Mean offshore TP concentrations ($\mu\text{gP L}^{-1}$) as a function of loads (per 1000 MTA) using the steady state response matrix (Table 2). See Table S-5 for the definitions of abbreviations and variables.

I.D.#	Equation	Eq. #
1.	$c^{SCR} = 6.2525 \cdot W_t^{SCR} = 6.2525 (W_s^{LH} + W_w^{SCR})$	(21)
2.	$c^{LSC} = 4.9245 \cdot (W_s^{LH} + W_w^{SCR} + W_w^{LSC} + 52.2 + 0.3)$	(22)
3.	$c^{DR} = 4.9035 \cdot (W_s^{LH} + W_w^{SCR} + W_w^{LSC} + 52.2 + 0.3) + 6.0535 \cdot W_w^{DR}$	(23)
4.	$c^{WB} = 2.5170 \cdot (W_s^{LH} + W_w^{SCR} + W_w^{LSC} + 52.2 + 0.3) + 3.1073 \cdot (W_w^{DR} + W_w^{WB} + 127.2 + 0.9)$	(24)
5.	$c^{CB} = 0.7321 \cdot (W_s^{LH} + W_w^{SCR} + W_w^{LSC} + 52.2 + 0.3) + 0.9038 \cdot (W_w^{DR} + W_w^{WB} + 127.2 + 0.9) + 1.4838 \cdot (W_w^{CB} + 624.0 + 4.5)$	(25)
6.	$c^{EB} = 0.3620 \cdot (W_s^{LH} + W_w^{SCR} + W_w^{LSC} + 52.2 + 0.3) + 0.4469 \cdot (W_w^{DR} + W_w^{WB} + 127.2 + 0.9) + 0.7322 \cdot (W_w^{CB} + 624.0 + 4.5) + 1.3063 \cdot (W_w^{EB} + 241.1 + 1.8)$	(26)

Numbers used throughout equations have the following meaning:

- 52.2, 127.2, 624.0 and 241.1, over-lake total atmospheric deposition of TP in MTA to Lake St. Clair, western basin, central basin, and eastern basin, respectively.
- 0.3, 0.9, 4.5, and 1.8, TP load in MTA with the direct ground water discharge into Lake St. Clair, western basin, central basin, and eastern basin, respectively
- All other numbers represent the equation coefficients

Table S-9. Steady state response matrix for the St. Clair Lake – Lake Erie continuum for the 2003- 2016 period. The values indicate the contributions to the mean annual TP concentration ($\mu\text{g P L}^{-1}$) in each segment (row) of the TP inflow from Lake Huron and those of the TP watershed load without accounting for the TP atmospheric deposition of the segment itself and any upstream segment. Abbreviations are the same as in Table 2.

Response	Watershed load							Total concentration
	LH	SCR	LSC	DR	WB	CB	EB	
St. Clair River (SCR)	11.64	1.30						12.94
Lake St. Clair (LSC)	9.17	1.02	4.49					14.68
Detroit River (DR)	9.14	1.02	4.47	4.03				18.66
Western Basin (WB)	4.68	0.53	2.30	2.07	9.38			18.96
Central Basin (CB)	1.37	0.15	0.67	0.60	2.73	3.77		9.29
Eastern Basin (EB)	0.68	0.07	0.33	0.30	1.35	1.86	1.90	6.49

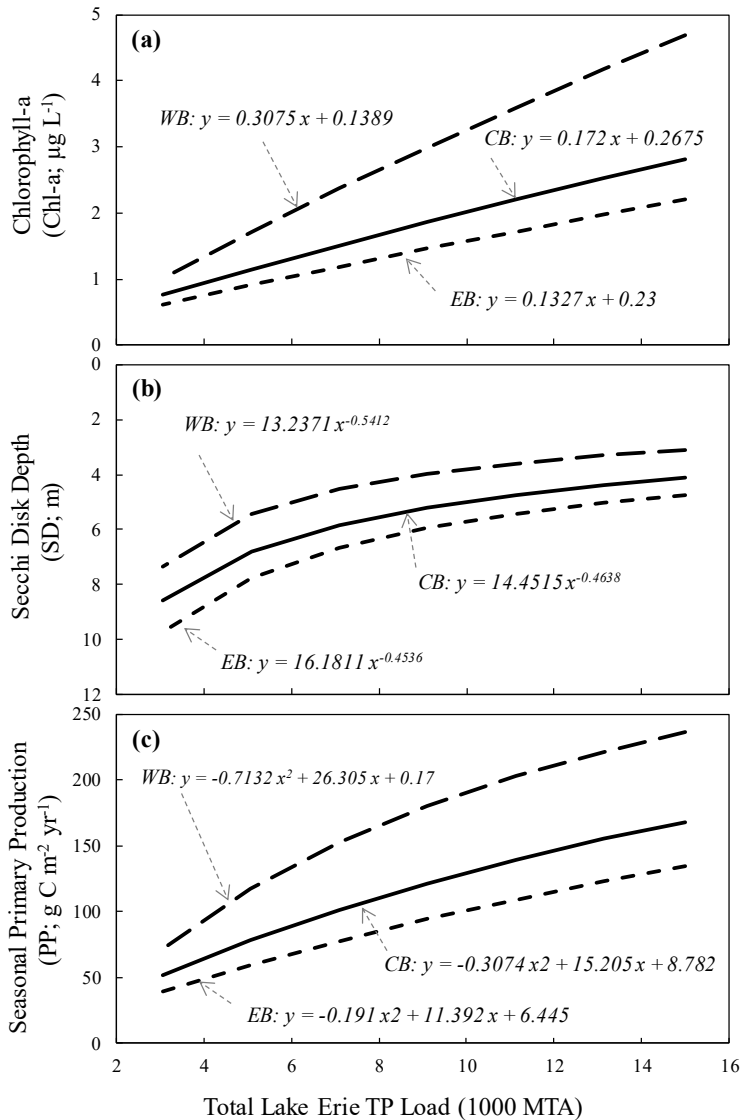


Figure S-1. Basin-specific load-response curves developed for Lake Erie for (a) summer Chlorophyll-a (Chl-a) concentration, (b) summer Secchi disk depth (SD), and (c) seasonal (May 1 – October 31) integral primary production by phytoplankton (PP) versus total Lake Erie total phosphorus (TP) loadings using literature reported relationships between spring TP concentration (Fig. 4b this study) and summer Chl-a and SD (Dove and Chapra, 2015), and between mean seasonal TP concentration (Fig. 4a this work) and seasonal (May 1 – October 31) PP (Millard et al., 1996; Graham et al., 1996). WB, CB, and EB mean western, central and eastern basins of Lake Erie, respectively.

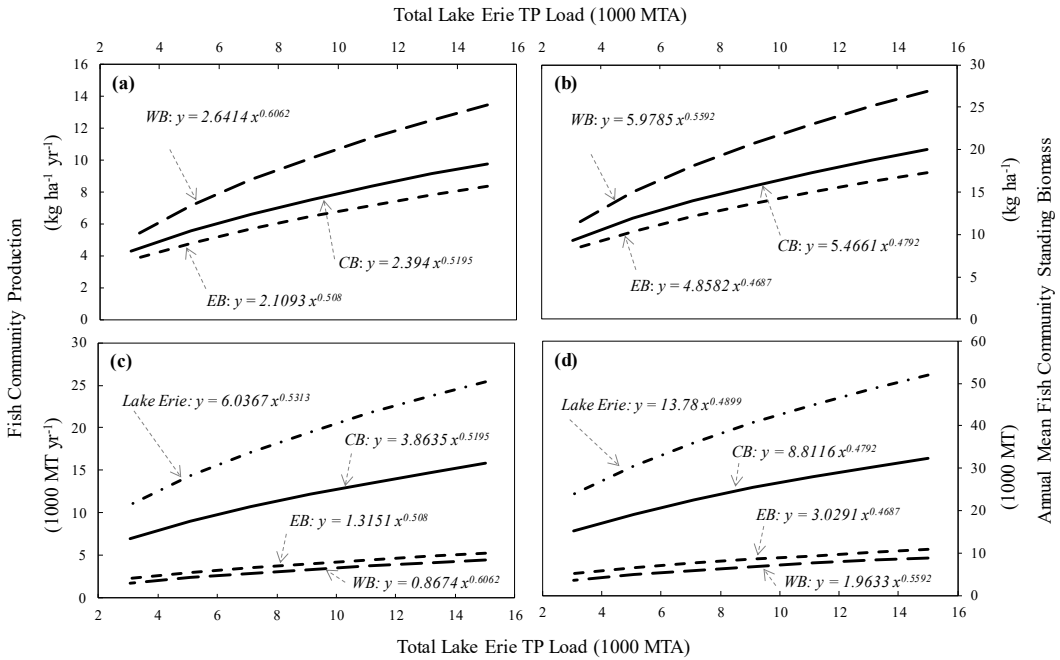


Figure S-2. Basin-specific and lake-wide (Lake Erie) load-response curves developed for Lake Erie for (a) fish community production (FP), and (b) for fish community standing biomass (FB) using global relationships between the season-averaged total phosphorus (TP) concentration (Fig. 4a this study) and FP and FB (Downing et al., 1990); (c) and (d) shows the same load-response curves as (a) and (b) but scaled to account for the entire basin or lake. WB, CB, and EB mean western, central and eastern basins of Lake Erie, respectively, while Lake Erie means entire lake.